Milthm ScoreV3 AccV2 Draft

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Contents

1 Introduction

This paper proposes the third-version scoring method (ScoreV3) and the second-version accuracy metric (AccV2) for the rhythm game "Milthm".

2 Terms and Definitions

2.1 List of Abbreviations

Abbreviation	Explanation
Milthm	the rhythm game "Milthm".
ScoreV3	the third-version scoring method.
AccV2	the second-version accuracy metric.

2.2 Glossary

Term	Explanation
Judgment point	A point within the judgment window that serves as the reference time to judge a specific player action.
Hit timing bias	The time difference, in milliseconds, between the player's action and the judgment point. Negative values indicate early hits; positive values indicate late hits.
Full Combo (FC)	For all judgment points, the player's actions produce judgments that continue the combo.
All Perfect (AP)	For all judgment points, the player's actions yield one of the highest-scoring judgment grades.
Theoretical Maximum (MAX)	For all judgment points, the player's actions yield the maximum possible reward.

2.3 Definitions

Definition 2.1 (Judgment Grade). *A judgment grade is a union of multiple continuous time intervals, used to describe the hit timing bias intervals.*

For a judgment grade l, we have

$$l = \bigcup_{i=1}^{n} I_i,$$

Here I_i is a continuous interval on $\mathbb{R}$.

Definition 2.2 (Judgment Set). *The judgment set $\mathcal{J}$ is a set whose elements are judgment grades; it also has a default element $m = \varnothing$ to represent that no judgment is produced.*

Property 2.2.1 (Disjointness). *For any $j_a, j_b \in \mathcal{J}$ with $j_a \neq j_b$, we have $j_a \cap j_b = \varnothing$.*

Property 2.2.2 (Interval Coverage)**.** $\bigcup_{j\in\mathcal{J}} j$ *is a continuous interval on* $\mathbb{R}$*.*

The judgment set $\mathcal{J}$ used by ScoreV3 is as follows:

Judgment Grade	**Name**	**Hit timing offset interval (ms)**
s	Exact	$[-35, 35]$
p	Perfect	$[-70, -35), (35, 70]$
g	Great	$[-105, -70), (70, 105]$
n	Good	$[-140, -105), (105, 140]$
b	Bad	$[-155, -140), (140, 155]$
m	Miss	$\varnothing$

Definition 2.3 (Judgment Mapping)**.** *The judgment mapping* $J : \mathbb{R} \to \mathcal{J}$ *maps the real line to the judgment set:*

$$J(\delta) = \begin{cases} j, & \exists j \in \mathcal{J}\left(\delta \in j\right), \\ m, & \textit{otherwise.} \end{cases}$$

Definition 2.4 (Sequence of Judgment Points)**.** *A sequence of judgment points* $\Sigma = \{\sigma_1, \sigma_2, \cdots, \sigma_N\}$ *is a finite, non-decreasing sequence, where each judgment point* $\sigma_i \in \mathbb{R}^*$*.*

Definition 2.5 (Hit-object Count)**.** *Strictly speaking, the hit-object count is not the same as the number of judgment points. In this paper, the hit-object count is defined as the number of judgment points in the usual sense.*

The hit-object count N *is the length of the sequence* Σ*, i.e.,* $N = |\Sigma|$*.*

Definition 2.6 (Hit Offset Sequence)**.** *The hit offset sequence* $\Delta = \{\delta_1, \delta_2, \cdots, \delta_N\}$ *is a sequence derived from the judgment point sequence and user actions, where* $|\Delta| = N$*, and each hit offset* $\delta_i \in \mathbb{R}$*.*

Definition 2.7 (Judgment Sequence)**.** *The judgment sequence* $\Xi = \{\xi_1, \xi_2, \cdots, \xi_N\}$ *satisfies* $\xi_i = J\left(\delta_i\right)$ *for each* i*.*

3 ScoreV3 Algorithm

3.1 Judgment Score

Definition 3.1 (Stepwise Judgment Score Function)**.** *The stepwise judgment score function* $S_{judge} : \mathcal{J} \to \mathbb{R}$ *maps the judgment set to real numbers.*

The stepwise judgment score function S_{judge} used in ScoreV3 is as follows:

Judgment Grade	**Score**
s	100,0000
p	99,0000
g	60,0000
n	30,0000
b	15,0000
m	0

Definition 3.2 (Average Judgment Score up to the n-th point)**.** *The average judgment score up to the n-th point AS_n is defined as:*

$$AS_n = \frac{\sum_{i=1}^{n} S_{judge}(\xi_i)}{n}.$$

3.2 Combo Compensation Multiplier

3.2.1 Base Compensation Multiplier

Definition 3.3 (Base Combo Score Mapping)**.** *The base combo score mapping $B : \mathcal{J} \times \mathbb{N}^+ \to \mathbb{R}^3$ maps the judgment grade ξ and the object count N to the combo base score upper bound $b_{\xi,N}$, combo base score lower bound $d_{\xi,N}$, and the judgment increment $a_{\xi,N}$.*

The base combo score mapping B used in ScoreV3 is as follows:

Judge ξ	**Upper Bound** $b_{\xi,N}$	**Lower Bound** $d_{\xi,N}$	**Increment** $a_{\xi,N}$
s	$\min(\max(\lfloor 0.24N \rfloor, 1), 192)$	0	2
p	$\min(\max(\lfloor 0.24N \rfloor, 1), 192)$	0	1
g	$\min(\max(\lfloor 0.16N \rfloor, 1), 128)$	0	0
n	$\min(\max(\lfloor 0.12N \rfloor, 1), 96)$	0	0
b	$\min(\max(\lfloor 0.10N \rfloor, 1), 80)$	0	0
m	$\min(\max(\lfloor 0.08N \rfloor, 1), 64)$	0	0

Definition 3.4 (Combo Score Sequence)**.** *The combo score sequence $\Theta = \{\theta_0, \theta_1, \cdots, \theta_N\}$ is a recurrence computed from the judgment sequence Ξ and the base combo score mapping B.*

The combo score sequence recurrence formula used in ScoreV3 is as follows:

$$\begin{aligned} \theta_0 &= b_{s,N}, \\ \theta_i &= \min\left(\max\left(\theta_{i-1} + a_{\xi_i,N}, d_{\xi_i,N}\right), b_{\xi_i,N}\right). \end{aligned}$$

Definition 3.5 (Maximum Combo Score)**.** *The maximum combo score b_{max} is the upper bound that the combo score can reach.*

In ScoreV3, $b_{max} = b_{s,N}$.

Definition 3.6 (Base Compensation Multiplier Sequence)**.** *The base compensation multiplier sequence* $\mathring{K} = \{\mathring{\kappa}_1, \mathring{\kappa}_2, \cdots, \mathring{\kappa}_N\}$ *describes the base compensation multiplier at specific judgment points. For the n-th judgment point, the base compensation multiplier* $\mathring{\kappa}_n$ *is calculated as follows:*

$$\mathring{\kappa}_n = \frac{\sum_{i=1}^{n} \theta_i}{n \cdot b_{max}}.$$

3.2.2 Compensation Multiplier Adjustment

Definition 3.7 (Maximum Combo Score Recovery Rate)**.** *The maximum combo score recovery rate* a_{max} *is the fastest achievable recovery increment among all judgment grades.*

In ScoreV3, $a_{max} = a_{s,N}$.

Definition 3.8 (Predicted Combo Score Sequence)**.** *The predicted combo score sequence* $\Theta^* = \{\theta_0^*, \theta_1^*, \cdots, \theta_N^*\}$ *describes the maximum combo score attainable at judgment point* i*. The relation between* Θ^* *and* Θ *is as follows:*

$$\theta_i^* = \max(\theta_{i-1} + a_{max}, 0).$$

Definition 3.9 (Combo Score Recovery Sequence)**.** *The combo score recovery sequence* $P = \{\rho_0, \rho_1, \cdots, \rho_N\}$ *describes the minimum number of judgment points required after the i-th judgment point to restore the combo score to its maximum value (excluding the i-th judgment point itself). The sequence* P *is related to the combo score sequence* Θ *as follows:*

$$\rho_i = \max\left(\left\lceil \frac{b_{max} - \theta_i}{a_{max}} - 1 \right\rceil, 0\right).$$

From Definitions 3.3 through 3.9, we see that the perturbation to the combo multiplier caused by identical judgments can vary with position. We therefore introduce the combo score compensation sequence to eliminate the positional effect on the multiplier.

Definition 3.10 (Combo Score Compensation Sequence)**.** *The combo score compensation sequence* $\hat{\Theta} = \left\{\hat{\theta}_0, \hat{\theta}_1, \hat{\theta}_2, \cdots, \hat{\theta}_N\right\}$ *is used to eliminate the positional effect of judgments on the base compensation multiplier. The specific calculation formula is as follows:*

$$\hat{\theta}_0 = 0,$$

$$\hat{\theta}_i = \begin{cases} \hat{\theta}_{i-1}, & i + \rho_i \leqslant N \text{ or } \theta_i = \theta_i^*, \\ \dfrac{(2 \cdot (b_{max} - \theta_i) - a_{max} \cdot (N + 1 - i + \rho_i)) \cdot (i + \rho_i - N)}{2}, & \text{otherwise.} \end{cases}$$

Definition 3.11 (Combo Compensation Multiplier up to the n-th point)**.** *The combo compensation multiplier up to the n-th point* κ_n *is defined as:*

$$\kappa_n = 0.4 + 0.6 \cdot \frac{\max\left(\left(\sum_{i=1}^{n} \theta_i\right) - \hat{\theta}_n, 0\right)}{n \cdot b_{max}}.$$

3.3 Combo Bonus

Definition 3.12 (Combo Judgment Mapping)**.** *The combo judgment mapping* $C : \mathcal{J} \to \mathbb{B}$ *maps the judgment set to the Boolean set* $\mathbb{B} = \{0, 1\}$*, where* 0 *denotes false and* 1 *denotes true.*

The combo judgment mapping C used by ScoreV3 is as follows:

$$C(\xi) = \begin{cases} 1, & \xi \in \{s, p, g, n\}, \\ 0, & \text{otherwise.} \end{cases}$$

Definition 3.13 (Current Combo Sequence)**.** *The current combo sequence* $\Gamma = \{\gamma_0, \gamma_1, \cdots, \gamma_N\}$ *describes the current combo count at judgment point* i*, with the following recurrence:*

$$\begin{aligned} \gamma_0 &= 0, \\ \gamma_i &= \begin{cases} \gamma_{i-1} + 1, & C(\xi_i) = 1, \\ 0, & C(\xi_i) = 0. \end{cases} \end{aligned}$$

Definition 3.14 (Running Maximum Combo Sequence)**.** *The running maximum combo sequence* $\Gamma^* = \{\gamma_0^*, \gamma_1^*, \cdots, \gamma_N^*\}$ *describes the highest combo count up to judgment point* i*, with the following recurrence:*

$$\begin{aligned} \gamma_0^* &= 0, \\ \gamma_i^* &= \max\left(\gamma_i, \gamma_{i-1}^*\right). \end{aligned}$$

Definition 3.15 (Combo Bonus up to the n-th point)**.** *The combo bonus up to the* n*-th point* CS_n *is defined as:*

$$CS_n = 5000 \cdot \frac{\gamma_n^*}{N}$$

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3.4 AP Bonus

Definition 3.16 (AP Bonus up to the n-th point)**.** *The AP bonus up to the* n*-th point* AP_n *is defined as:*

$$AP_n = \begin{cases} 5000 \cdot \dfrac{n}{N}, & \forall i \in \{1, 2, \cdots, n\} \left(\xi_i \in \{s, p\}\right), \\ 0, & \textit{otherwise.} \end{cases}$$

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3.5 Score Calculation

Definition 3.17 (Score up to the n-th point)**.** *The score up to the n-th point S_n is defined as:*

$$S_n = \kappa_n \cdot AS_n + CS_n + AP_n.$$

Definition 3.18 (Total Score)**.** *The total score for ScoreV3 is $TS = S_N$.*

Property 3.18.1 (Theoretical Maximum)**.** *The total score TS achieves the maximum value* 1,010,000 *if and only if* $\forall \xi \in \Xi \, (\xi = s)$.

4 AccV2 Algorithm

Definition 4.1 (Stepwise Accuracy Function)**.** *The stepwise accuracy function $A_{judge} : \mathcal{J} \to \mathbb{R}$ maps the judgment set to the real numbers.*

The stepwise accuracy function A_{judge} used by AccV2 is as follows:

Judgment Grade	**Accuracy**
s	100%
p	100%
g	60%
n	30%
b	15%
m	0%

Definition 4.2 (Accuracy up to the n-th point)**.** *The accuracy up to the n-th point ACC_n is defined as:*

$$ACC_n = \frac{\sum_{i=1}^{n} A_{judge}(\xi_i)}{n}.$$

Definition 4.3 (Overall Accuracy)**.** *The overall accuracy of AccV2 is $TA = ACC_N$.*

Property 4.3.1 (Maximum Accuracy)**.** *The total score TS achieves the maximum accuracy* 100% *if and only if* $\forall \xi \in \Xi \, (\xi \in \{s, p\})$.